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Technical Report

Rapport technique

1988 / 08

TR-ENG-001
NRC NO. 29808

FUEL FLOW MEASUREMENT IN GAS TURBINE TESTING

W. Grabe

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MESURES DU DÉBIT DU CARBURANT LORS DE L'ESSAI DES TURBINES À GAZ

W. Grabe

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TR-ENG-001
NRC No. 29808

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ACKNOWLEDGEMENTS

The author wishes to acknowledge the assistance given by many colleagues in the field, in particular, Dr. Ö. Gülder, Messrs. L. Gardner, P. Strigner, and G. Moroz, all from the Fuels and Lubricants Laboratory, and Mr. P. Turyk, Engine Laboratory, who was instrumental in the error analysis.

ABSTRACT

The accurate measurement of fuel flow constitutes an important part of gas turbine performance assessment. Prompted by an international engine testing program, and the mandate to improve all aspects of gas turbine performance evaluation, the Engine Laboratory has critically examined the existing fuel flow measuring methods. The report describes all elements of the fuel flow measuring system in detail, with source documentation wherever possible. Since the laboratory employs turbine flowmeters exclusively, the discourse has been limited to that type of flow establishment. Together with an in-house ballistic flowmeter calibrator, and an up-to-date data acquisition and reduction computer system, the present Engine Laboratory fuel flow measuring system provides gas turbine fuel flow assessment of known accuracy. An error analysis, for normal operating conditions, indicated an uncertainty in corrected fuel flow of ±0.75%. Recommendations have been made for improvements in some measurement components.

RÉSUMÉ

Le mesurage exact du débit du carburant constitue un facteur important dans l'évaluation du rendement de la turbine à gaz. Encouragé par un programme d'essai de moteurs à l'échelle internationale et mandaté pour améliorer tous les aspects de l'évaluation du rendement des turbines, le Laboratoire des moteurs a examiné d'un oeil critique les méthodes utilisées pour mesurer le débit du carburant. Le rapport décrit en détail tous les éléments du système de mesurage et présente des documents à l'appui lorsqu'il est possible de le faire. Étant donné que le Laboratoire emploie exclusivement des débitmètres à turbine, l'étude s'est restreinte à l'évaluation de ce type de débit. Épaulé par un calibrateur de débitmètre balistique maison et par un système informatique de pointe gérant l'acquisition et le dépouillement des données, le système actuel utilisé par le Laboratoire des moteurs pour mesurer le débit du carburant permet d'obtenir une évaluation du débit de la turbine d'une précision connue. Une analyse des erreurs, établie pour des conditions de fonctionnement normales, indique une incertitude dans le débit corrigé du carburant de ± 0.75%. Des recommandations visant à améliorer certains composants de mesure ont été formulées.

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FUEL FLOW MEASUREMENT IN GAS TURBINE TESTING

1.0 INTRODUCTION

In the accurate assessment of gas turbine performance, the establishment of correct fuel flow is important. Fuel flow is considered as a performance parameter on its own merits, but it also appears in the much used thrust-specific-fuel-consumption function. Thus the measurement of fuel flow is critical to the certification of new engines and the acceptance testing of overhauled ones.

Largely prompted by the Engine Laboratory's participation in the AGARD-sponsored Uniform Engine Testing Program, a reexamination of all performance data had been initiated, with the aim of improving the accuracy of data acquisition and reduction. The measurement of fuel flow was prominent in this upgrading program.

Although fuel flow, in aviation gas turbine testing, is evaluated in the form of mass flow, most measurements are being carried out with volume flowmeters, predominantly of the turbine type. The report will be restricted to the discussion of the various aspects of this type of flow measurement.

2.0 METHOD OF FUEL FLOW MEASUREMENT

Throughout the Engine Laboratory, the fuel flow measuring and calibration systems have been standardized to the products of one supplier. The general principle of turbine flowmeter application is well known; for an in-depth treatment see Zimmerman and Deery (1970). However, detailed methodology and several refinements are now possible.

Through calibration, a distinct relationship between volume flow through a meter and emitted pulses can be established (Section 3.0). This relationship is characterized by the K-factor as a function of the ratio of frequency to viscosity of the test fluid. The K-factor is the flowmeter's frequency divided by the flow, and is given in pulses per unit volume. For reasons of traceability and consistency, the US gallon has been retained as the unit of volume throughout. A typical calibration, consisting of several calibration runs, can been seen in Figure 1. Alternatively, the "litre" is being used as the measure of volume in the K-factor.

While efforts are made to convert to the SI system of units, imperial units are still widely used. Hence, equations in both types of units will be given.

The calculation of observed fuel flow, in conventional (imperial) units, takes the form:

$$WF = \frac{FREQ \cdot 8.3371616 \cdot RD(T_f) \cdot 3600}{CT \cdot K} \qquad (1)$$

where:

WF = observed fuel flow, lbm/h

FREQ = flowmeter frequency, pulses/s (Section 3.0)

8.3371616 = volume-to-mass conversion factor for pure, air-free water at standard temperature (Section 6.0)

$RD(T_f)$ = relative density (specific gravity) of the fuel at the temperature of flow measurement, $(lbm_{fuel,T_f}/ft^3)/(lbm_{H_2O,15.56°C}/ft^3)$ (Section 4.1)

CT = $1/(1+3\alpha(T_f-T_c))$ = flowmeter thermal correction factor (Section 5.0)

α = coefficient of thermal expansion of the material of the flowmeter body (for 304 stainless steel meters: 17.28×10^{-6} m/m°C with temperatures in °C)

T_f = observed fuel temperature, °C

T_c = average fuel temperature at flowmeter calibration, °C

K = flowmeter calibration factor, pulses/US gallon (Section 3.0)

In metric, or SI, units the fuel from equation becomes:

$$WF = \frac{FREQ \cdot 3.7816730 \cdot RD(T_f)}{CT \cdot K} \qquad (2)$$

where:

WF = fuel flow, kg/s
(to convert to g/s, multiply by a factor of 1000)

$RD(T_f)$ = relative density at temperature T_f, $(kg_{fuel,T_f}/\ell)/(kg_{H_2O,15.56°C}/\ell)$

Note: the numerical value does not change between systems

3.7816730 = volume-mass conversion factor (Section 6.0)

The units of the other variables remain unchanged from those of equation 1. The only non-metric unit is the "US gallon" of the K-factor. It was kept because it was the unit of calibration, both for the flowmeter as well as the Omnitrak calibrator.

To be able to compare fuel flows obtained under various engine inlet conditions, and also to account for different heats of combustion of fuels used, the observed fuel flows must be "corrected" to standard conditions:

$$WF_{CORR} = \frac{WF}{\delta\sqrt{\theta}} \cdot \frac{NHC}{NHCR} \qquad (3)$$

where

WF = fuel flow at test conditions lbm/h or kg/s

$$\delta = \frac{\text{engine inlet total pressure}}{\text{standard day barometric pressure}}$$

$$\theta = \frac{\text{engine inlet total temperature}}{\text{standard day temperature}}$$

NHC = net heat of combustion, units compatible with those of NHCR (Section 4.3)

NHCR = reference net heat of combustion, BTU/lbm or MJ/kg

The standard day barometric pressure can be given in imperial units as 14.696 psia or in metric units as 101.325 kPa. Consequently, the standard day temperature is 288.15 K (59°F or 15°C).

The reference value for the net heat of combustion (NHC) has been chosen by the Engine Laboratory as 42.80 MJ/kg, or 18400 BTU/lbm (Section 4.3). Other values may be specified for a particular program.

3.0 FLOWMETER CALIBRATION FACTOR

3.1 General

For accurate flow measurement, it is essential to calibrate turbine flowmeters, with subsequent periodic checks of the original calibration. These calibrations can be carried out by the manufacturer prior to delivery and also as follow-up service.

3.2 Flowmeter Calibrator

In 1984, the Engine Laboratory acquired its own Flow Technology Omnitrak ballistic flow calibrator, Model OT-150. The purchase was accompanied by a factory calibration. All in-house flowmeters were re-calibrated between four and seven times, to demonstrate repeatability of the Omnitrak calibrator, as well as of the turbine flowmeters, and to provide updated calibrations for the meters, within 0.05%.

In spring 1987, the Omnitrak itself was recalibrated according to the manufacturer's specifications and the recommendations of the Physics Division, NRC. The process will be documented in a

to-be-published report (Grabe and Carvish). Allowing for a minor correction in the water draw mass establishment, the calibration check agreed with the original calibration, validating the flowmeter calibrations carried out on the calibrator since its acquisition.

3.3 K-Factor

The sensitivity of a turbine flowmeter is given by its calibration factor, often called the K-factor. It is obtained from the ratio of meter frequency to volume flow through the meter:

$$K = \frac{FREQ \cdot 60}{WF} \quad \frac{Pulses}{US\ Gallon} \qquad (4)$$

where:

FREQ = frequency, pulses/s
WF = flow through the meter, US gal/min.

The flowmeter calibration, over its whole operating range, is obtained by plotting the K-factor against the meter frequency divided by the fluid's kinematic viscosity, ν. A typical example of such a calibration curve, or universal viscosity curve, is given in Figure 1. The large plot size was chosen for enhanced resolution.

As the calibration fluid for the Omnitrak, JP4 aviation turbine fuel was selected. This fuel was the most likely fluid to be measured by the flowmeters in the laboratory's gas turbine testing (Section 3.4).

All measured and derived meter calibration data are documented on a flowmeter calibration sheet, a sample of which is shown in Figure 2. This sheet is a modified version of the supplier's original data sheet, which was produced by the control console and line printer. The system, as purchased, was upgraded to include complete data reduction and to permit electronic data storage, instantaneous calibration data display on the console screen, retrieval of previous calibrations, as well as hard-copy printing of calibration plots. Information on this modification is available on request (Lopez, 1988). The sample data in Figure 2 are from one of the calibration jobs (87-34) of the plot given in Figure 1.

The most accurate way of obtaining the correct K-factor for a given flow condition has been found to be via a linearized look-up table. A sample table, the one of the calibration curve of Figure 1, can be seen in Figure 3. By placing the linear break points close together, especially in sections of high curvature, excellent simulation (±0.005% est.) of the smooth curve can be achieved. Automated curve fitting procedures have not been developed to date because of the complexity of the curves.

With few exceptions, calibration points have fallen within a band of ±0.05%. This is the value claimed by Flow Technology Inc. for the repeatability of the Omnitrak calibrator constant. The declared calibration performance of the Omnitrak related to the National Bureau of Standards (NBS), via transfer standard, is ±0.1% (FTI, 1982).

It should be noted that each flowmeter has its own distinct calibration characteristic, and, hence, individual calibrations are imperative. Furthermore, accurate flow measurement demands do not permit the use of an average K-factor over the range of the meter. However, turbine flowmeters perform reliably, in a repeatable manner, and provide a precise means of flow measurement, as long as certain provisions are made, as outlined below.

3.4 Operation and Installation Effects

For best results, certain care must be taken in the operation and installation of turbine flowmeters. It includes regular calibration checks, proper handling of the calibration curve, such as small-step linearization for a computer look-up table, the correct measurement of fuel temperature, and the establishment of the fuel viscosity at the temperature of the flow measurement. Last, but not least, good inflow and outflow conditions, to and from the meter, must be provided.

It is considered good practice to use the flowmeter in the same attitude in which it was calibrated, which is usually horizontal. While small differences, say 5°, may have no appreciable effect on measurements, larger differences in attitude could lead to inaccuracies of up to 3% (Shafer, 1961).

Flow profile distortions and swirl must be eliminated before entry into the meter. These disturbances may be caused by the proximity of a pump, sharp bends in the inlet pipe (Myles and Harrison, 1963), sudden reductions in the inflow pipe, and inner wall roughness. The most common way of improving the flow quality is by installing lengths of smooth, straight pipe fore and aft of the meter. While sources differ on the extension of these pipes, ten diameters upstream and five downstream are considered to be the minimum (FTI, SL-791). To counter high degrees of swirl, it may be necessary to install flow conditioners in the supply line. Several designs are commercially available, each with its own advantages* (Flow Factor, 1987).

Pulsations of a flow may increase meter readings and could cause bearing damage. It is recommended to keep pulsations below 10% by proper arrangements of the piping and components (FTI, SL-791).

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*A program is planned at the Engine Laboratory to assess the effects of a flow conditioner on flow measurement.

Flowmeters may be damaged by overspeeding at start-up, if admission of the fluid into the system is not built up gradually.

Since most flowmeters are used for one type of liquid in a given application, it serves no useful purpose to calibrate them with a variety of liquids. In order to achieve the highest degree of accuracy, precise calibrations with the liquid to be measured should be carried out (Shafer, 1961). This practice is followed by the Engine Laboratory where the calibration medium is JP4 turbo fuel. (One calibration, carried out by the flowmeter manufacturer, with three different fluids, showed no differences in the K-factors between 0.82 and 1.18 centiStokes (cSt), but a shift of about 0.2% with a fluid of 3.3 cSt viscosity.)

Finally, it should be remembered that most flowmeters are unidirectional and must be installed according to their appropriate markings.

The above comments give a limited outline of possible problem areas in the installation and operation of turbine flowmeters. The list is not exhaustive and papers on the subject, as well as manufacturers' manuals, should be consulted.

4.0 FUEL PROPERTIES

4.1 Relative Density

4.1.1 General

The relative density (RD), also called specific gravity (SG), of the fuel to be measured enters the flow calculation as a first order factor and is, hence, of prime importance. The establishment of relative density takes place in two steps: the evaluation of relative density, or density, at a given reference temperature of the fuel sample, and the correct assessment of the relative density at the fuel's temperature, at the point of flow measurement. Both phases will be discussed in detail below.

4.1.2 Relative Density at Standard Temperature

The relative density of a fuel is established by the Fuels and Lubricants Laboratory (F&LL), NRC at the accepted standard temperature for hydrocarbons, i.e. 60°F. The primary measurement taken is the API gravity, from which relative density (specific gravity) is derived. The relationship is

$$\text{API gravity, deg} = (141.5/\text{sp gr } 60/60^\circ\text{F}) - 131.5 \qquad (5)$$

(ASTM, 1984, D287)

where:

relative density 60/60°F (sp gr 60/60°F) =
(density of fuel, 60°F)/(density of water, 60°F) (6)

Note: 60°F = 15.56°C = 288.71 K

Several methods of relative density determination can be employed. They are documented in the Annual Books of ASTM Standards, Section 5 (ASTM, 1984). Of main significance, for this report, is their relative precision, since it affects the final accuracy of the fuel flow calculation.

One method regularly used by the F&LL is ASTM D287. Its precision is subdivided into repeatability and reproducibility*, their values being 0.2° and 0.5° API, respectively. For JP4 aviation fuel, with a nominal RD of 0.76, the repeatability translates into 0.0008 relative density, or 0.10%, and the reproducibility has a margin of 0.0020 relative density, or 0.27%.

An alternative method, based on the same hydrometer approach is ASTM D1298, which appears to be of higher accuracy. Fluids are divided into basic fluid types for precision assessment. For aviation fuels, classified as "non viscous" in this context, the repeatability of this method is 0.0005 in relative density, or 0.1° API gravity. The reproducibility is 0.0012 in relative density, or 0.3° API gravity. Again, for JP4 fuel, the repeatability becomes 0.066% and the reproducibility 0.16%. Thus, compared with method D287, method D1298 appears to have a better precision by nearly a factor of two, although the reasons for that are not understood, since the measurement processes are virtually identical.

For very precise establishment of relative density, a highly accurate method is given in ASTM D941, which uses a Lipkin Bicapillary Pycnometer. The density in grams/millilitre is calculated directly from the test results. Its repeatability is quoted as 0.0001 g/ml density, at a 95% confidence level, while its reproducibility is given as 0.0002 g/ml. With JP4 as the example, the claimed repeatability is within 0.013% and the reproducibility within 0.026%, or about an order of magnitude better than method D287. Unfortunately, the time required for this method is approximately 25 times that for method D287.

While the above comparison assesses the precision of the various methods available for determining relative density, or density, at standard temperature, other factors must be considered. They include the skill and thoroughness of the operator, the general conditions of the equipment and of the laboratory at large, and the quality of fuel sampling, among others. Hence, given the right circumstances, accurate results may be obtained from all methods, although the chances for that to happen are probably greatest with method D941. Thus, method ASTM D941 must be recommended for determining a fuel's relative density, whenever precise results are required.

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*The terms are defined in the ASTM Standards, Section 5, Vol. 05.01, D287, and also with other methods.

4.1.3 Relative Density Change with Temperature

Traditionally, the Engine Laboratory used a plot for estimating the fuel relative density (specific gravity), as a function of its temperature, which supposedly originated from the Royal Aeronautical Society/Institute of Petroleum [Fig. 4]. Since the lines are straight, but of slightly differing slope, the approach has been to select a line representing the mean value for the type of fuel under consideration. For example, the specification of Jet A-1 aviation fuel limits the 60/60°F (15.56/15.56°C) specific gravity between 0.776 and 0.840. Hence, a line for the mean value of 0.808 was drawn onto the plot [Fig. 4]. Subsequently, the mathematical expression for this line was derived and applied for all Jet A-1 fuel thermal expansion calculations. Specific gravities of JP4 fuels were handled in a like manner.

A brief, and far from exhaustive, survey of the methods employed by other agencies showed a great similarity amongst them [Table 1]. All expressions were linear, with somewhat differing coefficients of expansion. As a result, the RD_T lines for an assumed $RD_{15.56/15.56°C}$ of 0.81 fan out with decreasing temperatures [Fig. 5]. It appears that the linear expressions were based on that published by Barnett and Hibbard (1956). The latter also provided a curve relating the coefficient of thermal expansion, C_{ex}, to the specific gravity 60/60°F [Fig. 6].

While the Barnett-Hibbard equation is widely accepted, Chin and Lefebvre (1983) expanded it to take into account the effects of the fuel's critical temperature on density.

The modified equation for fuel density, as a function of temperature, is given as:

$$\rho_{FT} = \rho_{F288.6}\left[1 - 1.8C_{ex}(T - 288.6) - 0.090\,\frac{(T - 288.6)^2}{(T_{cr} - 288.6)^2}\right] \text{ kg/m}^3 \qquad (7)$$

where:

ρ_{FT} = density of the fuel at temperature T, kg/m³

$\rho_{F288.6}$ = density of the fuel at 288.6 K , kg/m³

C_{ex} = coefficient of thermal expansion, 1/°F
[See Fig. 6 or Tables 2 or 3]

T = temperature of the fuel at point of flow measurement, K

T_{cr} = critical temperature of the fuel, K [Table 3].

Since the relative density, formerly called specific gravity, at fuel temperature, T_f, is defined as:

$$RD_{FT} = \frac{\text{density } (T_f)}{\text{density of water } (288.71K)} = \frac{\rho_{FT}}{\rho_{H_2O,288.71\ K}} \qquad (8)$$

one can substitute ρ_{FT} from equation 7:

$$RD_{FT} = \frac{\rho_{F,288.6}}{\rho_{H_2O,288.71K}}\left[1 - 1.8\ C_{ex}(T-288.6) - 0.090\ \frac{(T-288.6)^2}{(T_{cr}-288.6)^2}\right] \qquad (9)$$

The universally accepted reference temperature being 288.71K (= 15.56°C = 60°F), equation 9 can be written in slightly modified form:

$$RD_{FT} = RD_{288.71}\left[1 - 1.8\ C_{ex}(T_f-288.71) - 0.090\ \frac{(T_f-288.71)^2}{(T_{cr}-288.71)^2}\right] \qquad (10)$$

This equation is now being used by the Engine Laboratory for calculating a fuel specific gravity, at a temperature other than standard.

Finally, some aspects of the fuel thermal expansion calculation will be commented on briefly.

As can be seen from Table 1, most agencies listed used a constant value for the coefficient of thermal expansion, regardless of the fuel's relative density. Depending on the value chosen and the temperature difference from the standard, this could lead to a noticeable error in the calculation of a fuel's density. In a test case, the fuel was assumed to be Jet A-1, at a temperature of -25°C. By using the coefficient of expansion corresponding to the mean relative density of 0.808, namely 0.00051, for the low limit relative density of 0.776, which should be 0.00055, an error of 0.3% was introduced. Hence, it is suggested that the proper value of the coefficient of thermal expansion, C_{ex}, be used from Figure 6. Distinct values of $RD_{60/60}$ and C_{ex} are given in Table 2, together with a cubic fit for the curve.

It will be noticed that the Chin-Lefebvre equation has the same form as the Barnett-Hibbard one, except for the correction term for the critical temperature of the fuel. Again, using Jet A-1, at -25°C, as an example, the calculated density would be 0.065% greater without the last term. Since the omission of this term would create a small but noticeable bias, the equation should be used in its entirety.

Chin and Lefebvre compared the values of measured densities of a selection of hydrocarbons, as a function of temperature, with predictions from their equation, and found excellent agreement [Fig. 7].

In conclusion, it is felt that the Chin-Lefebvre equation provides the best estimation of a fuel's relative density, as a function of its temperature.

4.2 Viscosity

A turbine flowmeter is sensitive to the viscosity of the fluid it measures. A full treatment of viscous effects on the meter's turbine blades is described by Zimmerman and Deery (1970).

As pointed out in Section 3, the calibration of a flowmeter is expressed by its calibration factor, K, as a function of the meter output frequency, divided by the fluid's kinematic viscosity. This calibration is also referred to as the universal viscosity curve. It collapses the calibrations obtained with several viscosities, of one or more types of fluids, into a unique curve. While the viscosity does not have a strong influence over most of the calibration range, viz. the flat section, it needs to be established with reasonable accuracy for measurements in the low flow range. (For the possible effects of different liquids see section 3.4.)

From a sample, the Fuels and Lubricants Laboratory determines the viscosity of the fuel to be measured, at a set of temperatures, e.g. -18, 0, 20, and 40°C, by employing ASTM method D445. These viscosities, if correctly established, will form a straight-line curve on an ASTM kinematic viscosity-temperature chart, a sample of which is given in Figure 8. For demonstration, the viscosity of a nominal fuel sample has been plotted. With this curve, the viscosity at any other operating temperature may be obtained by inspection.

This approach, however, is unsatisfactory when the data acquisition and reduction process is computerized. In that case, a computational method, found in D341 (ASTM, 1984), can be used. The kinematic viscosity-temperature relationship, for viscosities of less than 2.0 cSt is expressed as follows:

$$\log \log Z = A - B \log T \tag{11}$$

$$Z = \nu + 0.7 + \exp(-1.47-1.84\nu-0.51\nu^2) \tag{12}$$

$$\nu = [Z-0.7] - \exp(-0.7487-3.295[Z-0.7] + 0.6119[Z-0.7]^2 - 0.3193[Z-0.7]^3) \tag{13}$$

where:

$Z = f(\nu)$, as defined
T = fluid temperature, K or °R
ν = kinematic viscosity, cSt (= mm^2/s)
A,B = constants

From analysis, the viscosities of a fluid are established at two distinct temperatures, such that neither viscosity exceeds 2.0 cSt. Subsequently, equation 12 is solved for Z for the two viscosity values. With Z(T1), Z(T2) and T1, T2, the A and B constants can be found by two simultaneous equations [eq. 11]. Knowing the constants for the

particular fluid permits the calculation of Z for any desired temperature [eq. 11], and, finally, the computation of the kinematic viscosity ν by means of equation 13. The computation will yield accurate results below 2 cSt, and within 1% up to a viscosity of 5 cSt.

A computer program has been written, and is being used at the Engine Laboratory, which basically follows the above-described process, but employs a curve-fitting routine for obtaining the A and B constants from several viscosity-temperature pairs. The program is named VISCOS, and can be made available on request.

The ASTM Handbook gives the repeatability of method D445 as 0.35%, 19 times out of 20, and the reproducibility as 0.70%, with the same confidence. Depending on the slope of the calibration curve, at the point of reading, an error in viscosity may have zero effect, in the case of a horizontal line, or a maximum of less than 0.01%, taken from existing plots. Hence, the kinematic viscosity has negligible effect on the K-factor within the range of viscosities used in gas turbine testing.

4.3 Net Heat of Combustion

If one thinks of the engine fuel flow as an energy inflow into the system, the net heat of combustion (NHC), or lower heating value (LHV) as it is often called, must be taken into consideration. This is done in the form of a correction applied to the fuel flow equation (3). The correction will debit or credit a fuel flow for any heat of combustion other than the declared reference value.

Unfortunately, a universally accepted reference does not exist. One reference number for net heat of combustion that has been used by some agencies, among them the UK Ministry of Defence (Standard, 1987), and which has been adopted by the Engine Laboratory, is 42.80 MJ/kg (18480 BTU/lbm). This value is applied for all aviation fuels, regardless of type. It should be pointed out, though, that the choice of this reference may not have been a propitious one. A brief examination of fuel analysis data on hand at the Engine Laboratory has shown that all NHC values were above the reference one by between 1.0% and 2.3%. It appears that Canadian aviation fuels have a higher NHC than the average of other countries. This fact makes it imperative to quote the NHC reference value used when stating calculated corrected fuel flows.

The Fuel and Lubricants Laboratory estimates the net heat of combustion by the Standard Method described in Section D1405 (ASTM, 1984). It is based on the fuel's aniline point and API gravity, with a subsequent correction for sulfur content. The repeatability of this method is 0.012 MJ/kg (5 BTU/lb), while the reproducibility is quoted as 0.035 MJ/kg (15 BTU/lb). Translated into percentages, relative to 42.8 MJ/kg, the repeatability becomes 0.028% and the reproducibility 0.082%.

Other methods available are ASTM D240 and D2382, both of which use bomb calorimeters, and ASTM D3338, another computational method like D1405.

The ASTM Handbook emphasizes that the experimental measurements of heat of combustion are the basis of all heat determinations and that computational estimations are only approximate substitutes. Reviewing the calorimeter method D240, it is found that the repeatability is given as 0.13 MJ/kg (55 BTU/lb) and the reproducibility as 0.40 MJ/kg (175 BTU/lb). For a reference NHCR of 42.8 MJ/kg, these precision components translate into 0.30 and 0.93%, respectively.

The computational estimation of method D3338 is very similar to method D1405. It is based on the fuel's relative density or API gravity, its aromatics content, and its volatility. The repeatability is stated as 0.021 MJ/kg, or 8 BTU/lbm, and the reproducibility as 0.046 MJ/kg, or 20 BTU/lb. Using the above reference, the repeatability and reproducibility are 0.049 and 0.11%, respectively. In addition, for aviation fuels, this method is given an accuracy of 0.105 MJ/kg (45 BTU/lb), of measured heating values, with a 95% confidence level. For an NHCR of 42.8 MJ/kg, this accuracy comes to 0.25%.

Finally, a high-precision bomb calorimeter method is available, viz. ASTM D2382. The quoted repeatability of this method is 0.051 MJ/kg (22 BTU/lb) for jet fuel, or 0.12%, and the reproducibility is 0.130 MJ/kg (56 BTU/lb), or 0.30%, with respect to the same reference.

To facilitate comparison of the above precision values, a summary of them has been prepared in Table 4. While, at first glance, the computational estimations appear to provide values of considerably greater precision, this impression may be misleading. Considering that the calculated estimates are based on original tests with calorimeters, it is hard to see how their overall accuracy can be superior to those obtained by the primary methods of heat of combustion determination. While the computational methods may have high degrees of repeatability and reproducibility, their absolute accuracies could be decidedly lower.

In view of the above considerations, the bomb calorimeter method D2382 must be the first choice for net heats of combustion when great accuracy is required. If this method is unavailable, the computational one of D1405, at present used by the Fuels and Lubricants Laboratory, should suffice.

5.0 FLOWMETER THERMAL CORRECTION FACTOR

During a flow measurement process, the flowmeter will have a certain temperature, which can be approximated as being that of the fluid it measures. If this temperature differs from the one that prevailed at calibration, a temperature correction should be applied. It corrects for thermal expansion, or contraction, of the flowmeter's housing and turbine.

The Engine Laboratory adopted the formula recommended by the Instrument Society of America (ISA, 1977), which is:

$$K_o = \frac{K_c}{1 + 3\alpha(T_o - T_c)} = C_T K_c \qquad (14)$$

where

$$C_T = \frac{1}{1 + 3\alpha(T_o - T_c)} \qquad (15)$$

and C_T = thermal correction factor

K_o = calibration factor at operating temperature

K_c = calibration factor at calibrating temperature

α = thermal coefficient of linear expansion (in units corresponding to T_o,T_c)

T_o = operating temperature

T_c = calibration temperature (in same unit as T_o)

The thermal coefficient of expansion, α, for the NRC-used turbine flowmeters is $17.28 \cdot 10^{-6}$, with temperatures in °C.

The calibration temperature is somewhat harder to establish. All flowmeters have been calibrated several times, each time at a particular temperature. One approach would be to find the average calibration temperature for each meter and update it with every new calibration, since a calibration curve is based on all calibration points. This would be a cumbersome process, in light of the relatively small correction obtained from this effect. Instead, it was decided to take the average of all (average) calibration fluid temperatures for T_c. At the time of writing, the average of 66 calibration runs was 22.8°C (+4.9/-6.9°C). A previous averaging, with fewer points, had yielded 23.8°C, the number which is used in all flow calculations. While a difference of 1°C affects the correction factor by only 0.005%, it is nevertheless suggested to upgrade the T_c value from 23.8 to 22.8°C.

6.0 VOLUME-MASS CONVERSION FACTOR

Since the flowmeter under consideration is a volume measuring device, a volume-to-mass conversion is required. The calibration of all turbine flowmeters is in "pulses per US gallon", which is also the unit of the K-factor of the Omnitrak ballistic flowmeter calibrator. Hence, a relationship between US gallon and either pound (mass) or kilogram had to be established. Dealing with fluids, water at standard temperature was chosen for the conversion. The imperial, as well as the SI, factor, given in Section 2.0, will be derived below. It should be pointed out that it is very important to state a fluid's temperature at all times, if a high degree of accuracy is to be achieved.

In imperial units:

$$1 \text{ US gallon} = 1 \cancel{\text{US gallon}} \cdot 999.0123 \frac{\cancel{\text{kg}_{H_2O,15.56°C}}}{\cancel{\text{m}^3}} \cdot 2.2046226 \frac{\text{lbm}_{H_2O,15.56°C}}{\cancel{\text{kg}_{H_2O,15.56°C}}} \cdot 0.0037854118 \frac{\cancel{\text{m}^3}}{\cancel{\text{US Gallon}}}$$

$$1 \text{ US gallon} = 8.3371616 \text{ lbm}_{H_2O,15.56°C} \quad (16)$$

In metric units:

$$1 \text{ US gallon} = \cancel{1 \text{ US gallon}} \cdot 999.0123 \frac{\text{kg}_{H_2O,15.56°C}}{\cancel{\text{m}^3}} \cdot 0.0037854118 \frac{\cancel{\text{m}^3}}{\cancel{\text{US Gallon}}}$$

$$1 \text{ US gallon} = 3.7816730 \text{ kg}_{H_2O,15.56°C} \quad (17)$$

The physical relationships were obtained from Bigg (1967), and Zimmerman and Lavine (1961). The density used in equations 16 and 17 is for pure, air-free water.

The above conversion factors, when multiplied by the relative density (specific gravity) of the fluid measured, will provide the mass of fluid, at a given temperature, occupying the volume of one US gallon.

7.0 FUEL FLOW MEASUREMENT UNCERTAINTY

7.1 Observed Fuel Flow

In any precise engineering measurement, it is of great importance to know its accuracy, or tolerance. When dealing with a complex measurement, such as observed fuel flow, the task of establishing the final accuracy of the process is formidable. The uncertainty of a fuel flow measurement, taken at the prevailing test conditions, is affected by the uncertainties of all the elements in the process. While it is difficult to determine all the uncertainties, i.e. the precision and the bias, of each instrument, equipment, and process, an effort has been made to demonstrate the establishment of the accuracy of the laboratory's system. The details of the error analysis and the numerical error computation, using typical input values, are given in Appendix A. All values are specific for the nominal example values chosen. (A more general error analysis is intended for publication in the near future.)

First, a sample calculation was carried out with very severe, worst case values. The purpose was to establish an upper uncertainty limit for the fuel flow measuring process. The uncertainty calculated was ±0.69% [App. A]. Using more typical flow conditions, and a more precise analysis method for relative density, yielded an uncertainty of ±0.37%. This demonstrates that each measurement has its own uncertainty. While some input parameters are virtually constant, others are highly dependent on the flow conditions. However, it is believed that most uncertainties in observed fuel flow are well within ±0.75%.

The elements that most affect the uncertainty are, in order of magnitude:

- relative density
- K-factor

for the uncorrected fuel flow.

Upgrading of these elements would benefit the overall uncertainty of the fuel flow estimation the most. These relative effects on uncertainties could be significantly affected by large biases in the determination of fuel properties.

7.2 Corrected Fuel Flow

Before the fuel flow of a given test can be compared with flows of other tests, either in the same test setup or in other facilities, the observed fuel flow must be "corrected" to standard conditions [eq. 3]. This correction contains two elements, the inlet atmospheric conditions and the energy content of the fuel. Both of these components are based on measurements and, hence, contain uncertainties which must be added to the uncertainties of the observed fuel flow established in the previous sub-section.

Again, considering severe conditions first, the uncertainty of the corrected flow was calculated to be ±0.96%. Under more normal conditions, this uncertainty decreased to ±0.65% [App. A]. The largest contributors to uncertainty were ambient temperature and net heat of combustion.

While the above error analyses were the result of a conscientious effort to assess the uncertainties of all elements involved, as correctly as possible, some effects may have been underestimated or omitted altogether. However, the uncertainties of corrected fuel flows should be within ±0.75% for the majority of conditions and within ±1.0% even under the most arduous ones.

8.0 SUMMARY AND RECOMMENDATIONS

(i) A critical reassessment of the method for gas turbine fuel flow measurement yielded an upgraded system of estimable precision. This report gives a detailed description of all elements of the fuel flow calculation process.

(ii) The Engine Laboratory uses one type of volumetric flowmeters exclusively; these meters are calibrated regularly on an in-house Omnitrak ballistic calibrator. The calibrations produce a meter's K-factor, as a function of output frequency, which is the prominent parameter of the flow equation.

(iii) The fuel relative density, at standard temperature, is determined at the Fuels and Lubricants Laboratory (F&LL). It is recommended that ASTM method D941 be employed because of its high degree of precision.

(iv) For estimating the fuel thermal expansion, a relationship by Chin and Lefebvre (1983) was found to be the most precise one and has been adopted by the Engine Laboratory.

(v) Kinematic viscosity enters the fuel flow calculation as a weak function through the K-factor calibration. Fuel sample viscosities, at specific temperatures, are established by the F&LL. Viscosities at actual fuel temperatures are subsequently calculated according to the method given in ASTM D341.

(vi) The observed fuel flow is corrected for the fuel's net heat of combustion, with respect to a reference value. The F&LL calculates the net heat of combustion from the fuel constituents by the method of ASTM D1405. Because of its superior precision, it is recommended that the bomb calorimeter method of ASTM D2382 be used, if possible.

(vii) A small correction is computed for the thermal expansion of the flowmeter turbine and housing. A one degree change in temperature will affect a 0.005% change in correction factor. At extremely low temperatures, this correction becomes important.

(viii) Accurate volume-to-mass conversion factors, for the SI and imperial systems, were calculated.

(ix) For accurate flow measurement, certain precautions must be taken in installation and operation of a flowmeter. They include straight lengths of inlet and outlet piping, the possible use of flow conditioners, the correct placement of a meter in the system, and periodic calibration checks, preferably with the liquid to be measured in application.

(x) An effort was made to assess the cumulative error in fuel flow from the individual inaccuracies of the contributing components. Under average conditions, the uncertainty of corrected fuel flow was estimated to be within ±0.75% for most conditions, while under extreme border conditions the uncertainty should not exceed 1.0%.

(xi) Depending on the choice of reference value, the fuel flow correction for net heat of combustion may alter the observed flow by as much as 2.5%, or more. Hence, it is imperative to quote the reference net heat of combustion when presenting corrected fuel flow.

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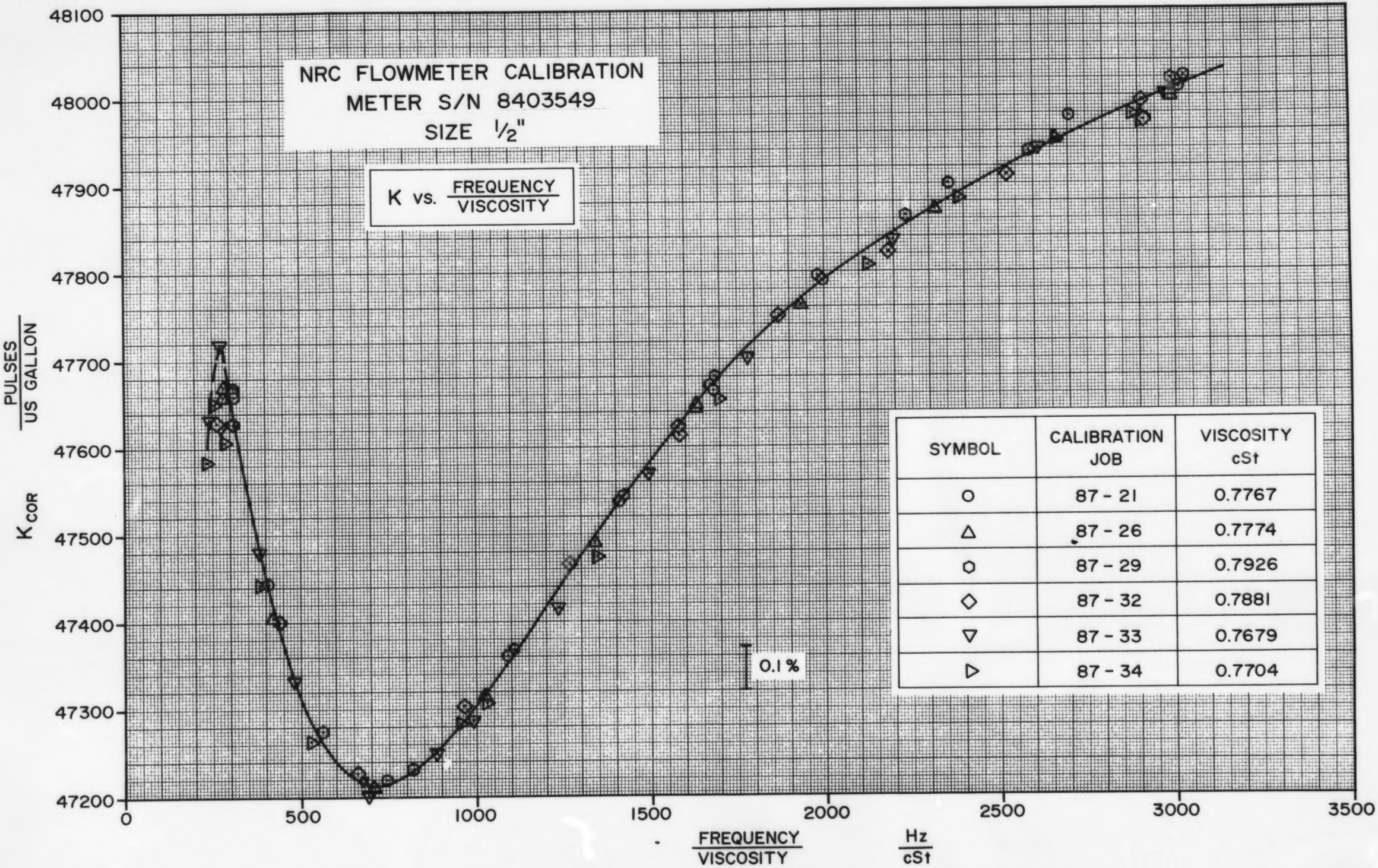

FIGURE 1: TYPICAL CALIBRATION OF A TURBINE FLOWMETER

NATIONAL RESEARCH COUNCIL CANADA
DIVISION OF MECHANICAL ENGINEERING

TURBINE FLOWMETER CALIBRATION

Customer: NRC-ENGINE LABORATORY
Date: 12-NOV-87 Job #: 87-34

CALIBRATOR DATA

Make: Flow Technology Omnitrak Calibrator System Model: OT-150 S/N: 83110022
Microtrak Constant: 62213 Volume Unit: U.S. Gallon
Calibration Fluid: JP4 Aviation Turbine Fuel
Specific Gravity(15.56/15.56 Deg.C): 0.7527
Fluid Temperature (Average)[Deg. C]: 22.90
Viscosity [cSt] : 0.7704
Room Temperature (Average)[Deg. C]: 23.50 Operating Pressure [Psig]: 80.00

FLOWMETER DATA

Make/Model : FT-4-8A3-LB Size : 1/2 IN.
S/N : 8403549 Pickoff Type: R.F.
Pickoff P/N: 27B 31199-101 Bearing Type: BALL

RUN	T VOL	T FMP	# FMP	VOL	FREQ	F RATE	Kcor	FREQ/VISC
1	2.3331	2.3336	4797.0	0.1	2055.6	2.5717	47951.	2668.1
2	2.1577	2.1577	4799.0	0.1	2224.1	2.7807	47981.	2886.8
3	2.9212	2.9214	4782.0	0.1	1636.9	2.0540	47808.	2124.6
4	2.6038	2.6042	4790.0	0.1	1839.3	2.3043	47884.	2387.4
5	3.6404	3.6401	4766.0	0.1	1309.3	1.6482	47655.	1699.4
6	4.5719	4.5716	4748.0	0.1	1038.6	1.3124	47475.	1348.0
7	6.4228	6.4222	4729.0	0.1	736.35	0.93417	47286.	955.74
8	5.9561	5.9553	4731.0	0.1	794.42	1.0074	47308.	1031.1
9	8.8675	8.8675	4722.0	0.1	532.51	0.67663	47212.	691.16
10	11.588	11.587	4727.0	0.1	407.94	0.51779	47263.	529.49
11	15.905	15.905	4745.0	0.1	298.34	0.37724	47442.	387.23
12	21.265	21.267	4762.0	0.1	223.92	0.28215	47608.	290.63
13	26.005	26.003	4759.0	0.1	183.02	0.23072	47585.	237.54
14	23.874	23.873	4766.0	0.1	199.64	0.25132	47653.	259.12
15	2.1400	2.1401	4798.0	0.1	2242.0	2.8037	47969.	2909.9
16	2.1361	2.1360	4798.0	0.1	2246.3	2.8089	47974.	2915.5

Fig.2 TYPICAL FLOWMETER CALIBRATION SHEET

T VOL=Time for Displaced Volume [Sec.]
T FMP=Time for Flowmeter Pulses [Sec.]
# FMP=Number of Flowmeter Pulses [ ]
VOL=Volume of Fluid Displaced [U.S. Gal.]
FREQ=Frequency of the Flowmeter [Pulses/Sec.]
F RATE=Flowrate [U.S. Gal./Min.]
Kcor=K-Factor of the Flowmeter Corrected to Standard Conditions [Pulses/U.S. Gal.]
FREQ/VISC=Frequency Divided by the Viscosity of the Fluid at the Temperature of the Run [Pulses/Sec.Cst.

Calibrated by:_____ Certified by:_____

Calibration of FTI Fuel Flowmeters

S/N: 1202357 Size: 3/4" JOB: _____

Linearized Look-Up Table

FREQ / VISC Hz / cSt	K_{corr} Pulses / U.S Gal		FREQ / VISC Hz / cSt	K_{corr} Pulses / U.S. Gal
2112	4370.0		425	4375.5
1800	4373.4		400	4374.0
1650	4375.0		370	4372.0
1550	4376.1		350	4370.5
1450	4377.1		325	4368.0
1300	4378.45		300	4365.25
1200	4379.2		292	4364.0
1100	4380.0		275	4361.6
1000	4380.6		267	4360.0
950	4380.9		250	4358.1
850	4381.2		230	4357.0
750	4381.35		213	4356.4
700	4381.3		200	4356.1
650	4381.1		175	4356.6
600	4380.75		168	4357.0
575	4380.4		163	4358.0
550	4380.0		141	4365.0
525	4379.4		120	4374.0
500	4378.6		100	4386.0
475	4377.75		90	4395.0
450	4376.65		80	4404.0

Figure 3. Sample Linearized Look-Up Table

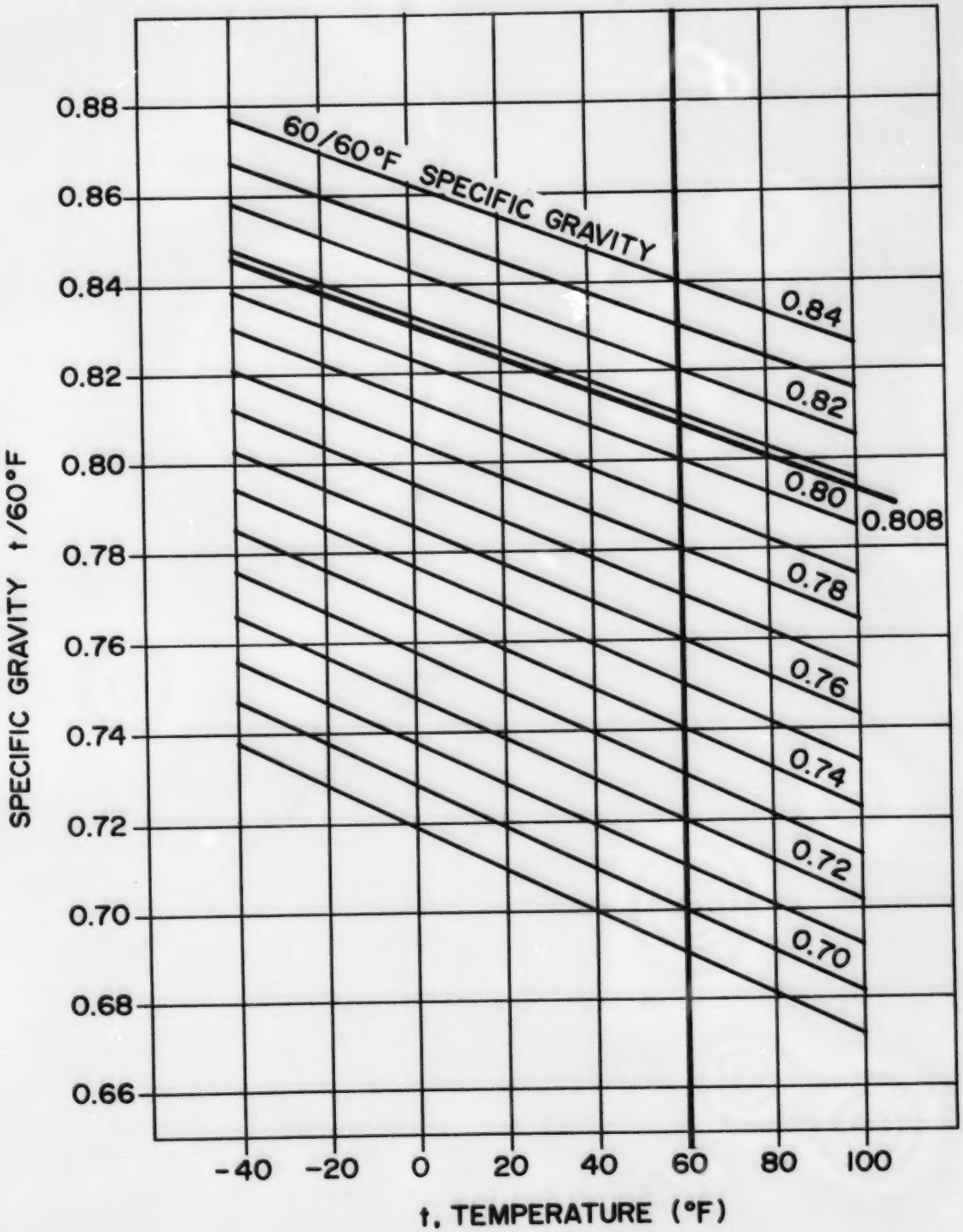

FIGURE 4: PETROLEUM PRODUCTS, CHANGES OF SPECIFIC GRAVITY WITH TEMPERATURE

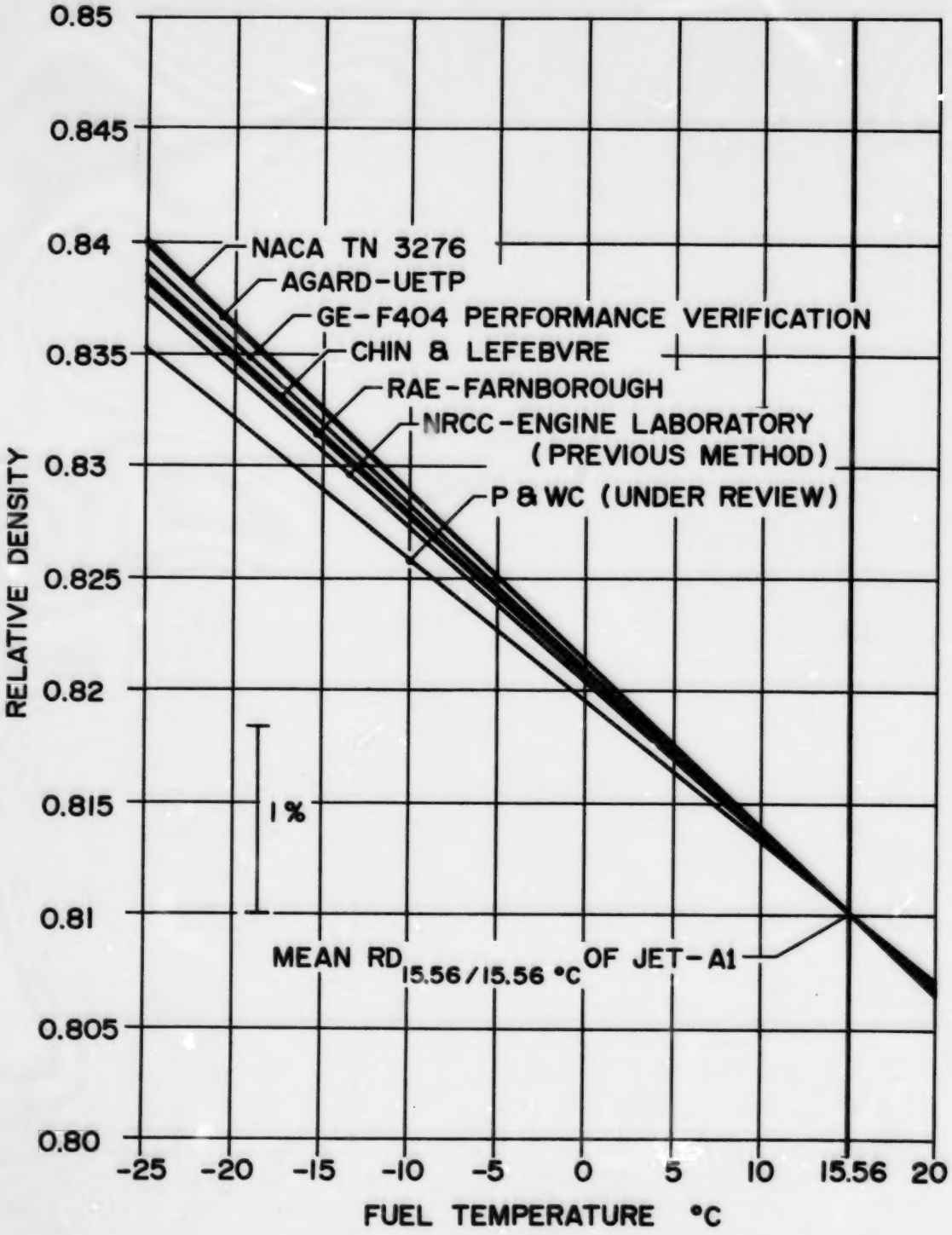

FIGURE 5: RELATIVE DENSITY VS. TEMPERATURE, COMPARISON BETWEEN AGENCIES

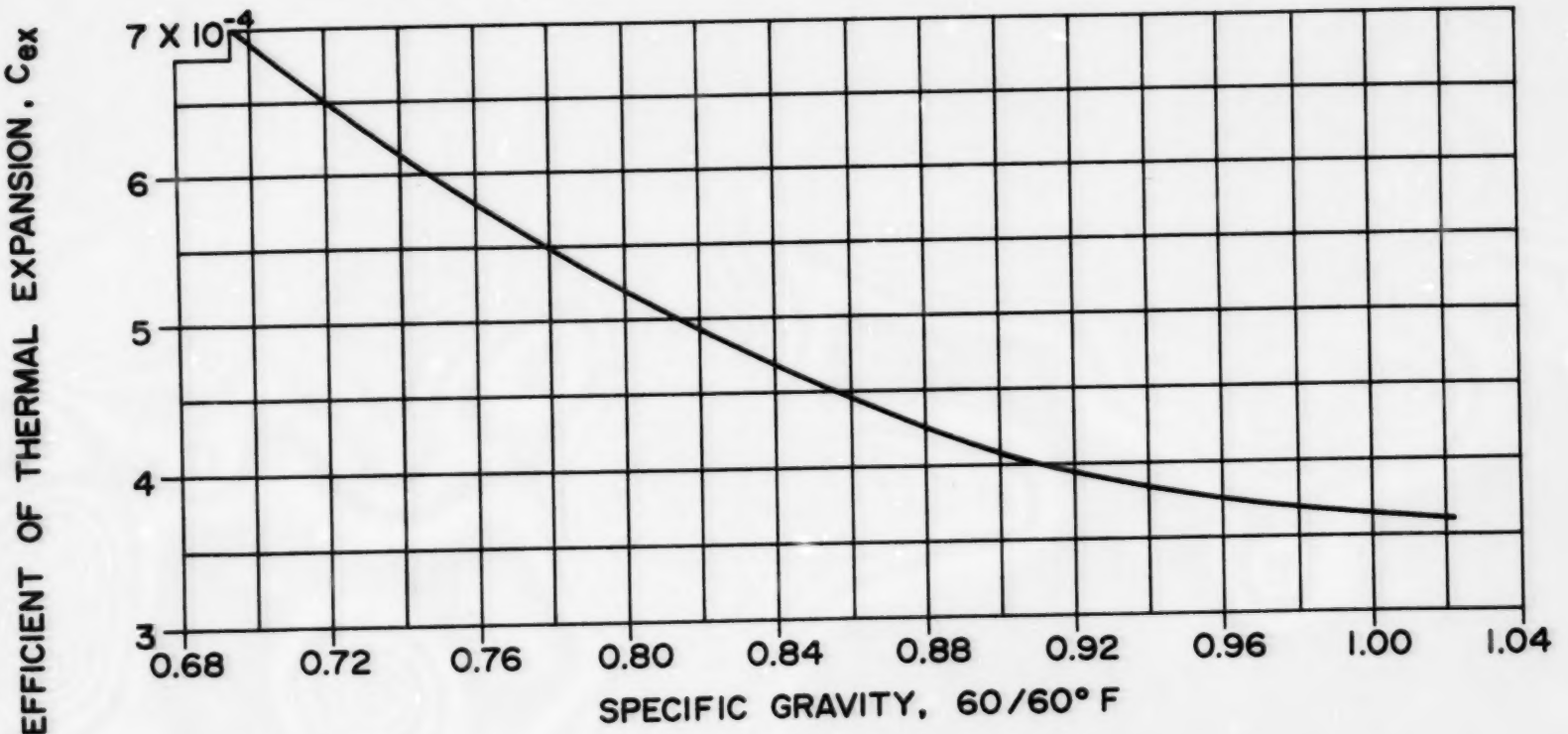

FIGURE 6: COEFFICIENTS OF THERMAL EXPANSION FOR FUELS OF DIFFERENT 60°F SPECIFIC GRAVITIES

REF. BARNETT AND HIBBARD, 1956

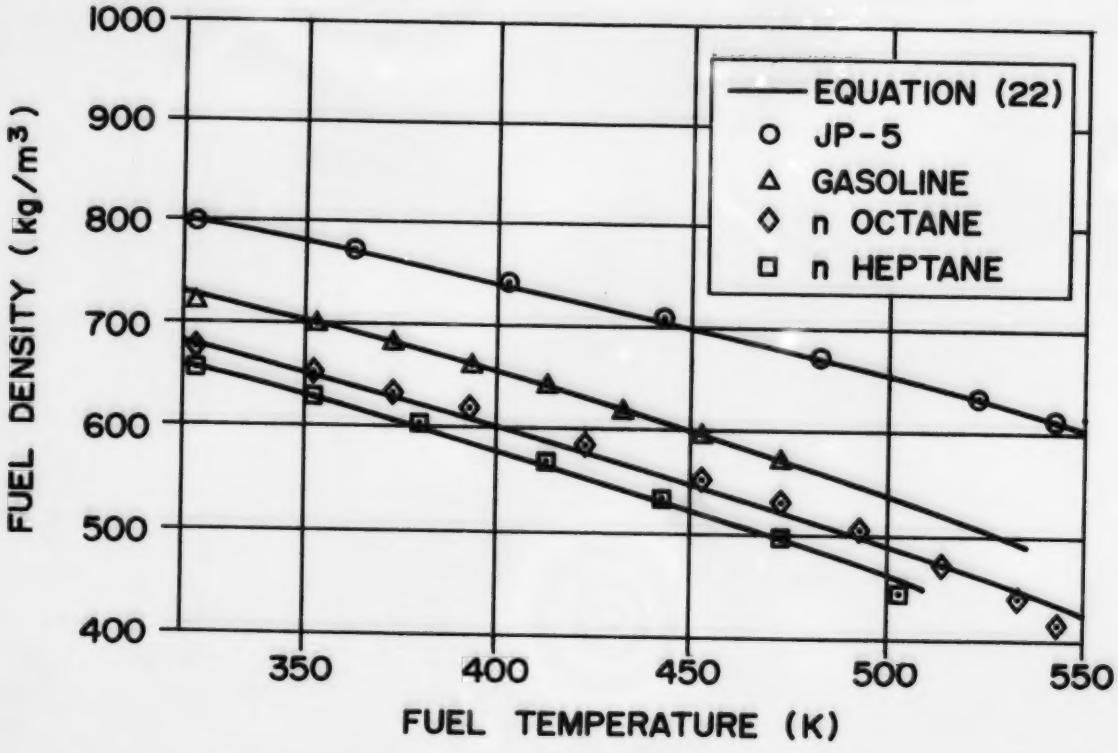

FIGURE 7: COMPARISON OF PREDICTED VALUES OF FUEL DENSITY WITH MEASURED VALUES

REF. CHIN AND LEFEBVRE, 1983.

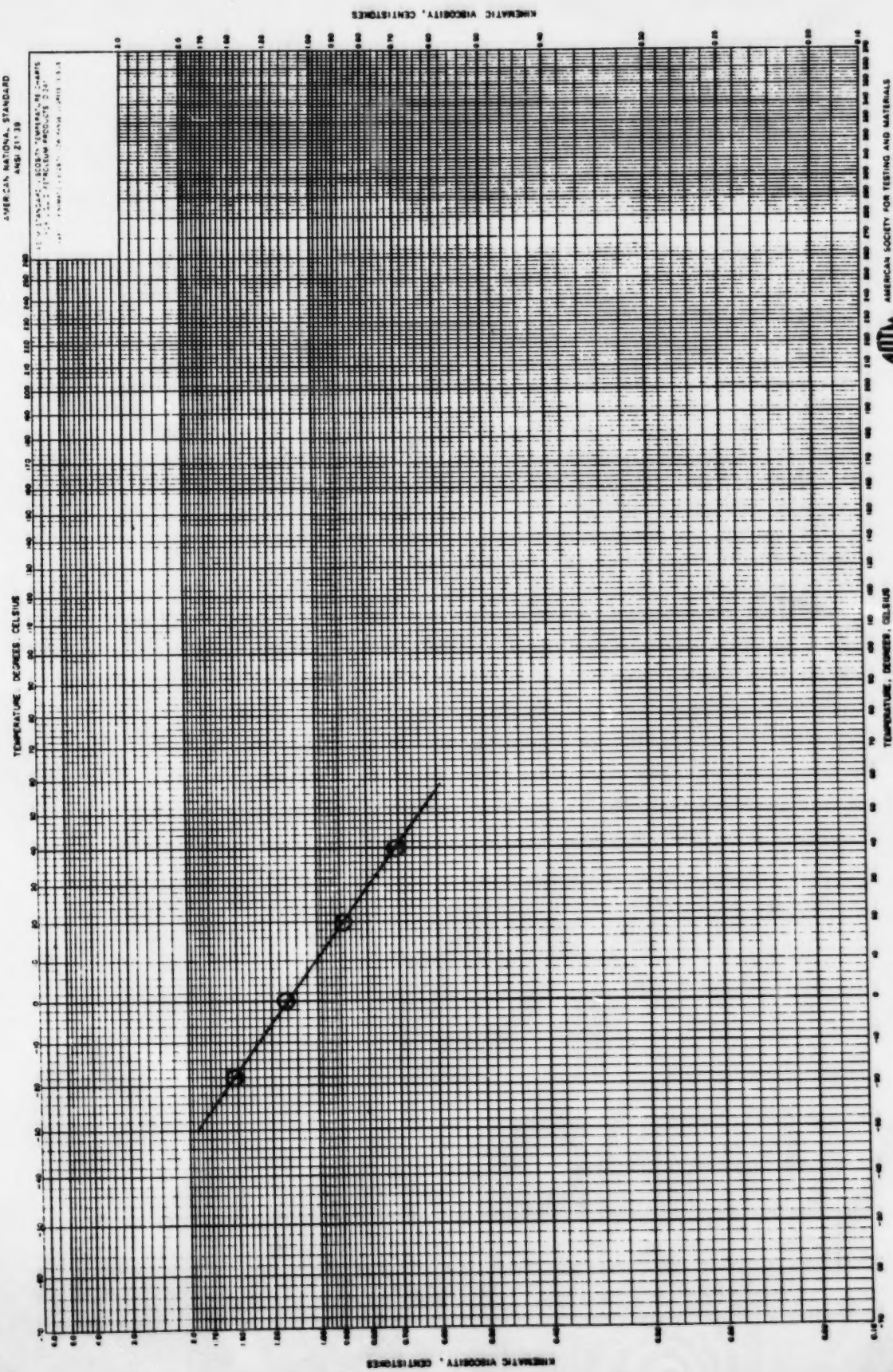
KINEMATIC VISCOSITY, CENTISTOKES
TEMPERATURE, DEGREES, CELSIUS
AMERICAN SOCIETY FOR TESTING AND MATERIALS
1916 RACE ST. PHILADELPHIA PA 19103

Fig. 8: KINEMATIC VISCOSITY OF JP4 SAMPLE 12/86 (ENGINE LABORATORY)

TABLE 1. SPECIFIC GRAVITY CALCULATION FROM A GIVEN $SG_{60/60}(RD_{15.56/15.56°C})$ AND T_{FUEL} COMPARISON BETWEEN AGENCIES

AGENCY/SOURCE	METHOD
NRCC (Previous Method)	$SG_{T_{JP4}} = SG_{60/60} + 0.01140 - 0.0007328 \cdot T[°C] = SG_{60/60} + 0.02443 - 0.0004071 \cdot T[°F]$ $SG_{T_{JETA1}} = SG_{60/60} + 0.1060 - 0.0006815 \cdot T[°C] = SG_{60/60} + 0.02271 - 0.0003786 \cdot T[°F]$
NASA TN 3276	$\rho_t = \rho_{60}\,[1 + C_{ex}(60 - t)]$; $C_{ex} = f(t)$ from curve; $t[°F]$
AGARD-UETP	$SG_{TF} = SG_{60}[1 + C_{ex}(288.7 - TWF)]$; $C_{ex} = 9.126 \cdot 10^{-4}$; $TWF[°C]$
GE-R79AEG096	$FSGWFT = FSG + 0.0004(TFSG - TWFT)$; $T[°F]$
RAE-FARNBOROUGH	$D_{T_{FUEL}} = D_{15°C} - 0.00071(T_{FUEL} - 15)$; $SG_{15°C} = D_{15°C}/0.999098$
P&WC *under review	$SG_{actual} = SG_{meas.}[1-0.00042833(T_{FUEL} - T_{SG})]$; $T[°F]$
CHIN & LEFEBVRE (Adopted by NRCC, modified)	$\rho_{FT} = \rho_{F_{288.6}}[1 - 1.8C_{ex}(T - 288.6) - 0.090 \frac{(T - 288.6)^2}{(T_{cr} - 288.6)^2}]$; $T[°C]$ T_{cr} = critical temperature of fuel

Table 2. EQUATION FOR COEFFICIENTS OF THERMAL EXPANSION
(DERIVED FROM FIGURE 6)

$RD_{15.56/15.56°C}$	$C_{ex_{curve}} \times 10^{-4}$	$C_{ex_{calc.}} \times 10^{-4}$	
0.70	6.93	6.929	
0.72	6.53	6.521	
0.73	6.33	6.329	
0.74	6.15	6.144	
0.75	5.96	5.967	
0.76	5.78	5.798	$C_{ex} = [52.3575$
0.77	5.62	5.635	$- 123.3954(RD_{15.56/15.56°C})$
0.78	5.47	5.480	$+ 104.7496 (RD_{15.56/15.56°C})^2$
0.79	5.33	5.331	$- 30.2584 (RD_{15.56/15.56C})^3]$
0.80	5.20	5.189	$\times 10^{-4}$
0.81	5.07	5.053	
0.82	4.94	4.923	With a coefficient of determination, R^2, of 0.9998
0.83	4.81	4.800	
0.84	4.69	4.682	
0.85	4.57	4.571	
0.86	4.45	4.464	
0.88	4.25	4.267	
0.90	4.075	4.090	
0.92	3.95	3.932	

Table 3. SOME THERMOPHYSICAL PROPERTIES
(FROM CHIN AND LEFEBVRE, 1983, UNLESS NOTED)

	JP4	JP5	JET-A1
T_{cr}, K	612.0	684.8	587*
DENSITY, $\frac{kg}{m^3}$	773	827	807**
C_{ex}, $^{\circ}F^{-1}$	0.000557	0.000485	0.000507***

* From O. Gulder, Fuels & Lubricants Laboratory
** Mean of specification limits
*** From Figure 6.

Table 4. SUMMARY OF PRECISION OF METHODS FOR NET HEAT OF COMBUSTION (NHC) DETERMINATION

METHOD		PRECISION					
Designation	Type	Repeatability			Reproducibility		
		MJ/kg	BTU/lb	%	MJ/kg	BTU/lb	%
ASTM[1] D240	Bomb Calorimeter	0.13	55	0.30	0.40	175	0.93
ASTM[1] D1405	Computational Estimation	0.012	5	0.028	0.035	15	0.082
ASTM[1] D2382	Bomb Calorimeter	0.051	22	0.12	0.13	56	0.30
ASTM[1,2] D3338	Computational Estimation	0.021	8	0.049	0.046	20	0.11

[1] 1984 Annual Book of ASTM Standards, Section 5, Vol. 05.01, 05.02, 05.03.

[2] Also the accuracy is given as 0.105 MJ/kg (45 BTU/lb), or 0.25%, with a 95% confidence level.

APPENDIX A

ERROR ANALYSIS OF FUEL FLOW MEASURING SYSTEM

In this Appendix, the complete uncertainty analysis for the fuel flow measurement system of the Engine Laboratory, NRC, will be delineated. By necessity, the classic approach (Abernethy, et al., 1973) could not be followed, because in most cases the precision, the bias, and the number of measurements were not available. Besides the development of general mathematical expressions, two sets of realistic conditions were chosen to demonstrate a range of uncertainties. One was rather severe in all its assumptions, while the other one depicted average conditions.

Conditions 1 (Severe):

a) Frequency, ω = 300 cps

b) Fuel Temperature, $T_f = -20°C$

c) Reproducibility of Relative Density Method
D1299 = ±0.16% (±0.0012)

d) Dynamic Viscosity (JP4,-20°C) = 1.4 cSt

e) Omnitrak Repeatability = ±0.05% (supplier)

f) Calibration Performance via
NBS Standards = ±0.1% (supplier)

g) Error in Look-up Table = ±0.005% (estimated)

Conditions 2 (Average):

a) Frequency, ω = 2200 cps (many calibration curves have a flatter slope in this regime)

b) Fuel Temperature = 10°C

c) Reproducibility of Relative Density Method
D941 = ±0.0002

d) Dynamic Viscosity (JP4,10°C) = 1.0 cSt

e) to g) Unchanged

(i) - Observed Fuel Flow

Observed fuel flow, based on turbine flowmeter measurements, is determined by equation 1 in the main text:

$$WF = \frac{FREQ \cdot 8.3371616 \cdot RD(T_f) \cdot 3600}{CT \cdot K} \qquad (1A)$$

Using abbreviated symbols and combining the constants, the equation can be rewritten:

$$W_f = \frac{C \cdot \omega \cdot \gamma}{C_T \cdot K} \qquad (2A)$$

where: $C = 8.3371616 \cdot 3600 = 30013.78176$, a conversion constant

ω = frequency (FREQ) $[\frac{cycles}{s}]$

γ = relative density of the fuel at temperature

T_f $(RD(T_f))$ $[-]$

$C_T = \frac{1}{1 + 3\alpha(T_f - T_c)}$ = correction factor for thermal expansion of flowmeter

$K = f(\omega/\nu)$ = calibration function $[\frac{pulses}{US\ gal}]$

ν = kinematic viscosity at T_f [cSt]

The accuracy in determining W_f will depend on the accuracy of the measurements of ω, γ, T_f, C_T, and K, i.e.:

$$dW_f = \left|\frac{\partial W_f}{\partial \omega}\right|\left|d\omega\right| + \left|\frac{\partial W_f}{\partial \gamma}\right|\left|d\gamma\right| + \left|\frac{\partial W_f}{\partial C_T}\right|\left|dC_T\right| + \left|\frac{\partial W_f}{\partial K}\right|\left|dK\right| \qquad (3A)$$

The partials of W_f with respect to all the quantities can be determined from the functional relationship given in equation 2A. The errors in the individual parameters are either known from specifications or are determined from other parameters on which they depend. Fuel flow depends implicitly on fuel temperature, which is taken into account through γ, C_T, and K.

Differentiating partially equation 2A one obtains:

$$\frac{\partial W_f}{\partial \omega} = \frac{C\gamma}{C_T K} \qquad (4A)$$

$$\frac{\partial W_f}{\partial \gamma} = \frac{C\omega}{C_T K} \qquad (5A)$$

$$\frac{\partial W_f}{\partial C_T} = -\frac{C\omega\gamma}{C_T^2 K} \qquad (6A)$$

$$\frac{\partial W_f}{\partial K} = -\frac{C\omega\gamma}{C_T K^2} \qquad (7A)$$

Fortunately, the fractional relationship of equation 2A lends itself to easy simplification. Dividing equation 3A by W_f:

$$\frac{|dW_f|}{W_f} = \frac{1}{W_f}\left|\frac{\partial W_f}{d\omega}\right||d\omega| + \frac{1}{W_f}\left|\frac{\partial W_f}{\partial \gamma}\right||d\gamma| + \frac{1}{W_f}\left|\frac{\partial W_f}{\partial C_T}\right||dC_T| + \frac{1}{W_f}\left|\frac{\partial W_f}{\partial K}\right||dK| \qquad (8A)$$

replacing

$$\frac{1}{W_f} \text{ by } \frac{C_T K}{C\omega\gamma} \text{ from equation 2A}$$

and taking the equivalences of the partial derivatives from equations 4A to 7A, equation 8A becomes:

$$\frac{|dW_f|}{W_f} = \frac{\cancel{C_T K}}{\cancel{C\omega\gamma}} \cdot \frac{\cancel{C\gamma}}{\cancel{C_T K}} \cdot |d\omega| + \frac{\cancel{C_T K}}{\cancel{C\omega\gamma}} \cdot \frac{\cancel{C\omega}}{\cancel{C_T K}} \cdot |d\gamma| + \frac{\cancel{C_T K}}{\cancel{C\omega\gamma}} \cdot \frac{\cancel{C\omega\gamma}}{C_T^{\cancel{2}} K} \cdot |dC_T| + \frac{\cancel{C_T K}}{\cancel{C\omega\gamma}} \cdot \frac{\cancel{C\omega\gamma}}{\cancel{C_T K^2}} |dK| \qquad (9A)$$

and

$$\frac{|dW_f|}{W_f} = \frac{|d\omega|}{\omega} + \frac{|d\gamma|}{\gamma} + \frac{|dC_T|}{C_T} + \frac{|dK|}{K} \qquad (10A)$$

This additive formulation will be more conservative in magnitude than the more usual root-sum-square method. The error in W_f can now be established by substituting the individual error values in terms of fractions of the nominal values. The determination of the parameter errors will be given below.

Frequency, ω

While at higher frequencies counters are very accurate, at lower frequencies the limiting factor is the resolution of 1 count. For multiple readings Abernethy, et al. (1973) suggest that this resolution is equivalent to a precision error of ±0.3 cps. This limitation, plus the high degree of curvature at the low end of the calibration curve make flowmeters inaccurate at the low range of their calibration. Inspection shows that the practical low-frequency limit varies from 200 cps for small flowmeters to 100 cps for the largest meters in the Engine Laboratory inventory. Therefore, using an average lower limit of 150 cps:

$$\frac{d\omega}{\omega} = \frac{0.3}{150} = 0.002 \quad \text{or} \quad 0.2\% \qquad (11A)$$

In most cases, the accuracy improves significantly with increasing frequencies. Data acquisition errors for frequencies are assumed to be negligible since the computer system clocks operate nominally at 1 MHz, with accuracies better than 0.01%.

Relative Density, γ

The functional relationship for γ is:

$$\gamma = \gamma_{15.56/15.56}\left[1-1.8\ C_{ex}(T_f-15.56) - 0.090\ \frac{(T_f-15.56)^2}{(T_{cr}-15.56)^2}\right] \qquad (12A)$$

(Assume this to be an exact relationship)

where: $\gamma_{15.56/15.56}$ = fuel relative density at 15.56°C(60°F) (for short: $\gamma_{15.56}$)

T_f = fuel temperature in °C

C_{ex} = critical temperature of the fuel in °C (a constant for a given fuel type).

The values for $\gamma_{15.56}$, T_f, and C_{ex} are variable, i.e. they are subject to error in their measurement. Hence, to find $d\gamma$ the total derivative must be found:

$$\gamma = f(\gamma_{15.56},\ T_f,\ C_{ex})$$

$$\therefore\ d\gamma = \left|\frac{\partial\gamma}{\partial\gamma_{15.56}}\right|\left|d\gamma_{15.56}\right| + \left|\frac{\partial\gamma}{\partial T_f}\right|\left|dT_f\right| + \left|\frac{\partial\gamma}{\partial C_{ex}}\right|\left|dC_{ex}\right| \qquad (13A)$$

All these quantities can be determined. The partial differentials are obtained from differentiating equation 12A and the total derivative from the measurement curves, or, in the case of C_{ex}, from a further determination of the derivative.

Thus

$$\frac{\partial\gamma}{\partial\gamma_{15.56}} = 1-1.8\ C_{ex}(T_f-15.56) - 0.090\ \frac{(T_f-15.56)^2}{(T_{cr}-15.56)^2} \qquad (14A)$$

$$= f_1 \qquad (15A)$$

$$\frac{\partial\gamma}{\partial T_f} = \gamma_{15.56}\left[-1.8C_{ex}-2(0.090)\frac{(T_f-15.56)}{(T_{cr}-15.56)^2}\right]$$

$$= -\gamma_{15.56}\left[1.8C_{ex}+0.180\ \frac{(T_f-15.56)}{(T_{cr}-15.56)^2}\right] \qquad (16A)$$

$$\frac{\partial\gamma}{\partial T_f} = -\gamma_{15.56}\ f_2 \qquad (17A)$$

$$\frac{\partial\gamma}{\partial C_{ex}} = \gamma_{15.56}\left[-1.8(T_f-15.56)\right]$$

$$= -1.8\gamma_{15.56}(T_f-15.56) \qquad (18A)$$

Now, $C_{ex} = f(\gamma_{15.56})$

$$C_{ex} = [52.3575 - 123.3954\ \gamma_{15.56} + 104.7496\ \gamma^2_{15.56} - 30.2584\ \gamma^3_{15.56}] \cdot 10^{-4} \quad (19A)$$

(Assume this equation to be exact.)

Thus $$dC_{ex} = \frac{\partial C_{ex}}{\partial \gamma_{15.56}}\, d\gamma_{15.56} \quad (20A)$$

$$dC_{ex} = \partial\gamma_{15.56}[-123.3954+(2)(104.7496)\gamma_{15.56} - (3)(30.2584)\gamma^2_{15.56}] \cdot 10^{-4}$$

$$= d\gamma_{15.56}[-123.3954+209.4992\ \gamma_{15.56} - 90.7752\ \gamma^2_{15.56}] \cdot 10^{-4} \quad (21A)$$

$$dC_{ex} = d\gamma_{15.56}\ f_3 \quad (22A)$$

Substituting equations 15A, 17A, 18A, and 22A into equation 13A, and using absolute values:

$$d\gamma = |d\gamma_{15.56}||f_1| + |dT||-\gamma_{15.56}\ f_2| + |d\gamma_{15.56}||-1.8\gamma_{15.56}(T_f-15.56)||f_3|$$

$$d\gamma = |d\gamma_{15.56}|(|f_1| + 1.8\gamma_{15.56}|T_f-15.56||f_3|) + |dT||\gamma_{15.56}f_2| \quad (23A)$$

where f_1, f_2, and f_3 are defined in equations 14A, 16A, and 21A, respectively.

For a sample calculation some extreme conditions were chosen in order to obtain near-maximum error limits (p.A1).

Let $\gamma_{15.56} = 0.761 \pm 0.16\%$ (reproducibility for method D1298)

$= 0.761 \pm 0.0012$

and $T_f = -20°C \pm \epsilon$

where ϵ = total uncertainty of the temperature measuring system

= t/c error + voltage/engineering units (V/EU) bias + voltage engineering (V/EU) units precision

For premium grade wire (ANSI MC 96.1-1975):

t/c error = ±0.5°C

For the NRC system:

V/EU bias = ±0.089%
V/EU precision = ±0.044%

The first two terms are biases which must be added by root-mean-square:

$$\epsilon = \pm \left| \pm \sqrt{0.5^2 + (0.00089 \cdot 268)^2} \right| + \left| (\pm 2 \cdot 0.00044 \cdot 268) \right|$$

$$\epsilon = \pm 0.79\ ^\circ C \text{ for } T = -20^\circ C\ (268K)$$

$$T_f = -20^\circ \pm 0.79^\circ C$$

$$T_{cr} = 612K = 339^\circ C \quad \text{for JP4}$$

from equation 19A:

$$C_{ex} = 5.7811 \cdot 10^{-4}$$

from equation 14A:

$$f_1 = 1.03591$$

from equation 16A:

$$f_2 = 0.0009794$$

from equation 21A:

$$|f_3| = |-16.5363 \cdot 10^{-4}| = 0.001654$$

$$d\gamma_{15.56} = \pm 0.0012 \quad \text{(method D 1298)}$$

$$dT = \pm 0.79^\circ C$$

Substituting into equation 23A

$$d\gamma = |0.0012[1.03591 + 1.8(0.761)(|-35.56|)(0.001654)] + 0.79(0.761)(0.0009794)|$$

$$d\gamma = 0.001929 \qquad (25A)$$

Integrating equation 15A yields:

$$\gamma = \gamma_{15.56} f_1 = 0.761(1.03591)$$

$$\gamma = 0.7883 \qquad (26A)$$

and $$\frac{d\gamma}{\gamma} = \frac{0.001929}{0.7883} = 0.002447 \qquad (27A)$$

thus $\gamma = 0.7883 \pm 0.002447$ or

$$\gamma = 0.7883 \pm 0.310\%$$

Note that this relative density and its uncertainty were based on the assumed conditions 1.

Correction Factor, C_T

The correction factor for the thermal expansion of the flowmeter housing and turbine is given by:

$$C_T = \frac{1}{[1+3\alpha(T_f-T_c)]} \tag{28A}$$

where $\alpha = 17.28 \cdot 10^{-6}$ in $\frac{1}{^oC}$

(provided by manufacturer, stainless steel 304)

T_f = fuel temperature, in °C

T_c = fluid temperature at calibration, in °C

Differentiating partially:

$$\frac{\partial C_T}{\partial T_f} = \frac{3\alpha}{-[1+3\alpha(T_f - T_c)]^2} = f_4 \tag{29A}$$

and

$$\frac{\partial C_T}{\partial T_c} = \frac{-3\alpha}{-[1-3\alpha(T_f - T_c)]^2} = f_5 \tag{30A}$$

$$dC_T = |dT_f||f_4| + |dT_c||f_5| \tag{31A}$$

Using the same fuel temperature as before, viz. $T_f = -20°C$, and taking the average of all fluid temperatures at calibration, $T_c = 22.8°C$ with the appropriate tolerances:

$T_f = -20° \pm 0.79°C$, i.e. $dT = \pm 0.79°C$

$T_c = 22.8 \pm 5°C$ (variation in calibration temperature),

i.e. $dT_c = \pm 5.0°C$

Replacing in equation 31A

$$dC_T = |\pm 0.79| \left| \frac{3 \cdot 17.28 \cdot 10^{-6}}{-[1+3 \cdot 17.28 \cdot 10^{-6}(-20-22.8)]^2} \right| + |\pm 5.0| \left| \frac{-3 \cdot 17.28 \cdot 10^{-6}}{-[1+3 \cdot 17.28 \cdot 10^{-6}(-20-22.8)]^2} \right| \tag{32A}$$

$$dC_T = 0.0003015 \tag{33A}$$

$$C_T = \frac{1}{[1+3 \cdot 17.28 \cdot 10^{-6}(-20-22.8)]}$$

$$C_T = 1.002224 \tag{34A}$$

and

$$\frac{dC_T}{C_T} = \frac{0.0003015}{1.002224}$$

$$\frac{dC_T}{C_T} = 0.0003008 \tag{35A}$$

Calibrating Factor, K

The determination of the K-factor is perhaps the most difficult of the parameters since a functional relationship of the calibration curves exists only in graphical form (for a typical sample see Figure 1).

The calibration points are scattered within a band, as the result of uncertainty in the values obtained for K and the uncertainty in ω and ν.

$$K = f(\omega, \nu, OC)$$

where OC is the uncertainty in K which stems from the error in the calibration of the Omnitrak calibrator.

Then $$dK = \frac{\partial K}{\partial \omega} d\omega + \frac{\partial K}{\partial \nu} d\nu + \frac{\partial K}{\partial OC} dOC \qquad (36A)$$

However, it would be more useful to cast this equation into a form which reflects the functional dependence of K on ω, and ν, i.e. K as $f(\omega/)$,

Hence

$$K = f(\omega/\nu, OC)$$

and $$dK = \left| \frac{\partial K}{\partial(\omega/\nu)} \right| \left| d(\omega/\nu) \right| + \left| \frac{\partial K}{\partial OC} \right| \left| dOC \right| \qquad (37A)$$

Note that the error in K is dependent on the fuel temperature, T_f, only indirectly through ν, and thus is not explicitly included in the functional dependence.

The uncertainty in the individual measurements of ω and ν are derived from

$$d(\omega/\nu) = \left| \frac{1}{\nu} d\omega \right| + \left| -\frac{\omega}{\nu^2} d\nu \right| \qquad (38A)$$

The elements for determining $d(\omega/\nu)$ are known:

$d\omega = 0.3$ from equation 11A

$$d\nu = \frac{\partial \nu}{\partial T} dT + \text{reproducibility} \qquad (39A)$$

where the second term is obtained from the method used in establishing the viscosity, viz. ASTM D445, which has a reproducibility of 0.70%.

While a complex functional relationship exists, one can perhaps simplify the viscosity's dependence on temperature, for aviation fuel, within a range of expectable temperatures, by

$$\frac{\partial \nu}{\partial T} \approx \left(\frac{\Delta \nu}{\Delta T}\right)_{AVE} = -0.012 \quad cST/°C$$

and $dT = \pm 0.79°C$ from equation 24A

Then equation 39A becomes:

$$d\nu = |-0.012||\pm 0.79| + 0.0070 \cdot \nu \qquad (40A)$$

At the assumed fuel temperature of condition 1, -20°C, the dynamic viscosity of the sample fuel [Fig. 8] is 1.4 cSt.

Placing this value in equation 40A

$d\nu = 0.0193$ cST.

The term $\frac{\partial K}{\partial OC}\, dOC$ represents all of the uncertainties in the actual value of K. These include:

Omnitrak - repeatability = ±0.05%
- calibration performance related to NBS via transfer standard = ±0.1%

Error in look-up table = ± 0.005% (est.)

If the repeatability and the error in look-up table are considered to be precision indices and the calibration performance is a bias limit then:

$$S = \sqrt{0.0005^2 + 0.00005^2} = 0.0005025$$

$B = 0.001$

Uncertainty, $U = \pm (0.001 + 2 \cdot 0.0005025)$

$U = \pm 0.002005$ or ±0.2%

This uncertainty is actually:

$$\frac{dK_{OC}}{K_{OC}} = \frac{\partial K}{\partial OC}\, dOC = 0.002000 \quad \text{or} \quad 0.20\%$$

Finally, the total error in the K-factor, dK, can be calculated by substitution in equation 37A, divided by K:

$$\frac{dK}{K} = \frac{1}{K} \left| \frac{\partial K}{\partial(\omega/\nu)} \right| \left[\left| \frac{d\omega}{\nu} \right| + \left| \frac{\omega}{\nu^2}\, d\nu \right| \right] + \left| \frac{dK_{OC}}{K_{OC}} \right| \qquad (42A)$$

The element unknown is the partial derivative of the K-factor plot, $\partial K/\partial(\omega/\nu)$. While this value will vary from plot to plot, and form one part of the calibration curve to the other , a rather severe example will be chosen:

From the calibration plot of flowmeter S/N 840 3549 [Fig. 1]:

$$\frac{\partial K}{\partial(\omega/\nu)} \approx \frac{\Delta K}{\Delta(\omega/\nu)} = \frac{(47700-47620)}{(290-320)} = -2.667\ \frac{\text{Pulses} \cdot \text{s} \cdot \text{cSt}}{\text{US Gal} \cdot \text{Pulses}} \qquad (43A)$$

This partial derivative describes a steep slope of the curve, on the low end of the range. Almost always, this value decreases at higher flows, because of the flattening of the calibration curve, and it may approach zero.

To complete the sample calculation for the severe condition:

with $\nu = 1.4$ cSt (sample fuel at -20°C)

$\omega = 305$ cps

$K = 47660 \frac{\text{Pulses}}{\text{US Gallon}}$ } mean values from equation 43A

Substituting in equation (42A)

$$\frac{dK}{K} = \frac{1}{47660} \left| -2.667 \left| \left[\left| \frac{0.3}{1.4} \right| + \left| \frac{305}{1.4^2} 0.0193 \right| \right] \right| +0.00200 \right| \quad (44A)$$

$$\frac{dK}{K} = \pm 0.002180 \quad (45A)$$

Fuel Flow

All elements of the uncertainty train have now been established for the calculation of the total error in fuel flow.

Recalling equation 10A and the example values from equations 11A, 27A, 35A, and 45A:

$$\frac{|dW_f|}{W_f} = |\pm 0.002| + |\pm 0.002447| + |\pm 0.0003008| + |\pm 0.002180|$$

$$\frac{dW_f}{W_f} = \pm 0.00693 \quad \text{or} \quad \pm 0.693\% \quad (46A)$$

The uncertainty presents rather severe conditions. Under most conditions, and with the use of improved analysis methods, the fuel flow uncertainty could be greatly reduced.

Using conditions 2 (p. A1):

at $\omega = 2200$ cps:

$$\frac{d\omega}{\omega} = \frac{0.3}{2200} = 0.0001364 \quad (47A)$$

using method D941:

$$\frac{d\gamma}{\gamma} = 0.001117 \quad (48A)$$

at $\omega = 2200$ cps

$$\frac{\partial K}{\partial \omega / \nu} = 0.240 \quad (49A)$$

The differential dK/K becomes:

$$\frac{dK}{K} = 0.002189 \quad (50A)$$

Thus the fuel flow uncertainty for the assumptions of condition 2 will be:

$$\frac{|dW_f|}{W_f} = |0.0001364| + |0.001117| + |0.0003019| + |0.002189|$$

$$\frac{dW_f}{W_f} = \pm\ 0.003744 \quad \text{or} \quad \pm\ 0.374\% \qquad (51A)$$

(ii) - Corrected Fuel Flow

Observed fuel flow is corrected to standard conditions as per equation 3 in the main text.

$$W_{f_{cor}} = \frac{W_f}{\delta\sqrt{\theta}} \cdot \frac{NHC}{NHCR} \text{ (glossary in Section 2.0)} \qquad (52A)$$

This correction reduces the observed fuel flow to standard day atmospheric conditions and provides a common basis of net heat of combustion. It may be said that the second correction factor changes the "fuel flow" into an "energy flow", which is ultimately the objective in a thermodynamic study.

For the purpose of the error analysis, equation 52A can be simplified in terms of its variables:

$$W_{f_{cor}} = \frac{W_f\ NHC}{P_2\ \sqrt{T_2}} \qquad 53A)$$

where P_2 = compressor inlet total pressure

T_2 = compressor inlet total temperature

Reference conditions for T_2, P_2, and NHCR are assumed to be exact.

Similar to the analysis of the observed fuel flow uncertainty, the accuracy of the corrected flow depends on the measurement accuracies of its constituent elements: W_f, NHC, P_2, and T_2.

Then $$W_{f_{cor}} = f(W_f,\ P_2,\ T_2,\ NHC)$$

and $$dW_{f_{cor}} = \frac{\partial W_{f_{cor}}}{\partial W_f} dW_f + \frac{\partial W_{f_{cor}}}{\partial P_2} dP_2 + \frac{\partial W_{f_{cor}}}{\partial T_2} dT_2 + \frac{\partial W_{f_{cor}}}{\partial NHC} dNHC \qquad (54A)$$

Differentiating equation 53A partially:

$$\frac{\partial W_{f_{cor}}}{\partial W_f} = \frac{NHC}{P_2\ \sqrt{T_2}} \qquad (55A)$$

$$\frac{\partial W_{f_{cor}}}{\partial P_2} = \frac{-W_f \ NHC}{P_2^2 \sqrt{T_2}} \tag{56A}$$

$$\frac{\partial W_{f_{cor}}}{\partial T_2} = -\frac{1}{2} \frac{W_f \ NHC}{P_2 \sqrt{T_2^3}} \tag{57A}$$

$$\frac{\partial W_{f_{cor}}}{\partial NHC} = \frac{W_f}{P_2 \sqrt{T_2}} \tag{58A}$$

Dividing equation 54A by $W_{f_{cor}}$, but substituting $\frac{1}{W_{f_{cor}}}$ by its equivalent $\frac{P_2 \sqrt{T_2}}{W_f \ NHC}$ on the RHS, and substituting the partial derivatives by their values of equations 55A to 58A, one obtains:

$$\frac{\partial W_{f_{cor}}}{\partial W_f} = \frac{P_2 \sqrt{T_2}}{W_f \ NHC} \cdot \frac{NHC}{P_2 \sqrt{T_2}} \cdot dW_f + \frac{P_2 \sqrt{T_2}}{W_f \ NHC} \cdot \frac{-W_f \ NHC}{P_2^2 \sqrt{T_2}} \cdot dP_2 + \frac{P_2 \sqrt{T_2}}{W_f \ NHC} \cdot \frac{-W_f \ NHC}{2P_2 \sqrt{T_2^3}} \cdot dT_2 + \frac{P_2 \sqrt{T_2}}{W_f \ NHC} \cdot \frac{W_f}{P_2 \sqrt{T_2}} \cdot dNHC$$

and

$$\frac{\left| d \ W_{f_{cor}} \right|}{W_{f_{cor}}} = \frac{|dW_f|}{W_f} + \frac{|-dP_2|}{P_2} + \frac{|-dT_2|}{2T_2} + \frac{|dNHC|}{NHC} \tag{59A}$$

Each parameter term can be evaluated from its own uncertainty, either computed or from known tolerances.

The uncertainty in observed fuel flow has been determined above [eq. 46A]:

$$\frac{dW_f}{W_f} = \pm 0.00693 \text{ or } \pm 0.693\% \tag{60A}$$

(severe assumptions)

The pressure uncertainty is given by

$$dP_2 = |\pm 0.00032| + 2|\pm 0.00005| \qquad (61A)$$

where 0.00032 = bias

0.00005 = precision

for precision flowmeters used in the laboratory

For the standard atmospheric pressure of 101.33 kPa:

$$\frac{dP_2}{P_2} = \frac{0.00042 \cdot 101.33}{101.33} = \pm 0.00042 \text{ or } \pm 0.042\% \qquad (62A)$$

The uncertainty of temperature measurements has been determined above as:

$$dT = dT_2 = \pm 0.81^{\circ}C \text{ (Std. Day)} \qquad (63A)$$

For a standard day, the temperature uncertainty becomes:

$$\frac{dT_2}{T_2} = \frac{\pm 0.81}{288.15} = \pm 0.00281 \quad \text{or} \quad \pm 0.281\% \qquad (64A)$$

The method for establishing the net heat of combustion used by the F&LL gives a repeatability of 0.028% and a reproducibility of 0.082% (based on JP4 fuel). Choosing the reproducibility as a measure of uncertainty:

$$\frac{dNHC}{NHC} = \pm 0.00082 \quad \text{or} \quad \pm 0.082\% \qquad (65A)$$

Replacing in equation 59A:

$$\frac{dW_{f_{cor}}}{W_f} = |\pm 0.00693| + |\pm 0.00042| + |\pm 0.00140| + |\pm 0.00082|$$

$$\frac{dW_{f_{cor}}}{W_{f_{cor}}} = \pm 0.00957 \quad \text{or} \quad \pm 0.957\% \qquad (66A)$$

Exchanging the uncertainty in observed fuel flow for the one calculated for average conditions, viz.

$$\frac{|dW_f|}{W_f} = 0.003744 \quad \text{(from equation 51A)}$$

without changing the uncertainties of the correction parameters, would yield a new corrected fuel flow uncertainty for average conditions of:-

$$\frac{dW_{f_{cor}}}{W_f} = \pm 0.00638 \quad \text{or} \quad \pm 0.638\%.$$

REPORT DOCUMENTATION PAGE/PAGE DE DOCUMENTATION DE RAPPORT

REPORT/RAPPORT	REPORT/RAPPORT
1a TR-ENG-001	1b NRC No. 29808
SECURITY CLASSIFICATION/ CLASSIFICATION DE SÉCURITÉ 2 Unclassified	DISTRIBUTION/DIFFUSION ☐ Controlled/Contrôlé ☒ Unlimited/Illimité 3

TITLE/SUBTITLE/TITRE/SOUS-TITRE

Fuel Flow Measurement in Gas Turbine Testing

4

AUTHOR(S)/AUTEUR(S)

W. Grabe

5

SERIES/SÉRIE

6

CORPORATE AUTHOR/PERFORMING AGENCY/AUTEUR D'ENTREPRISE/AGENCE D'EXÉCUTION

7 National Research Council Canada Engine Laboratory
Division of Mechanical Engineering

SPONSORING AGENCY/AGENCE DE SUBVENTION

8

DATE	FILE/DOSSIER	LAB. ORDER COMMANDE DU LAB.	PAGES	DIAGS	REFS
9 1988/08	10	11	12a 41	12b 8	12c 14

NOTES

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DESCRIPTORS(KEY WORDS)/MOTS-CLÉS

fuel flow, data acquisition, reduction computer system, ballistic flowmeter

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SUMMARY/SOMMAIRE

The report describes all elements of the Engine Laboratory fuel flow measurement system.

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